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CP violation in the secluded $\mathrm{U}(1)'$-extended MSSM

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Abstract: We study the Higgs sector of the secluded $\mathrm{U}(1)'$-extended MSSM (sMSSM) focusing on CP violation. Using the one-loop effective potential that includes contributions from quarks and squarks in the third generation, we search for the allowed region under theoretical and experimental constraints. It is found that the possible region for the electroweak vacuum to exist is quite limited, depending on the parameters in the model. The masses and couplings of the Higgs bosons are calculated with/without CP violation. Even at the tree level, CP violation is possible by complex soft SUSY breaking masses. Similar to the CPX scenario in the MSSM, the scalar-pseudoscalar mixing enables the lightest Higgs boson mass to become smaller than the Z boson mass while the coupling with the Z boson is sufficiently suppressed to avoid the LEP experimental constraints. However, unlike the CPX scenario, large μ and A are not required for the realization of large CP violation. The typical spectrum of the SUSY particles is thus different. We also investigate the possible upper bound of the lightest Higgs boson in the case of spontaneous CP violation. The maximal value of it can reach above 100 GeV with maximal CP-violating phases.

Keywords: Higgs Physics, Supersymmetric Standard Model, CP violation.

Contents

1. Introduction

Many new physics models have been proposed to address the issue of the so-called gauge hierarchy problem that cannot be resolved within the framework of the standard model (SM). Supersymmetric extensions of the SM have been paid much attention as possible solutions to this problem. In particular, the minimal supersymmetric standard model (MSSM) can solve not only this problem but also cosmological problems such as dark matter and baryon asymmetry of the Universe and so on. Nevertheless, the model still has an unattractive feature: the μ problem, where μ appears in the mass term of the higgsinos. As long as no special symmetry exist in the theory, the scale of μ is supposed to be the grand unified theory (GUT)/Planck scale from the naturalness point of view. However, once the electroweak symmetry is broken, the scale of μ should be at about the W boson mass. One direction to provide a natural scale for μ is to introduce a gauge singlet field (S) into the MSSM. Several variations of this extension have been proposed: the next-to-MSSM (NMSSM) [1 – 3], the nearly MSSM (nMSSM) [4, 5], the $\mathrm{U}(1)'$-extended MSSM (UMSSM) [6 – 8], and the secluded $\mathrm{U}(1)'$-extended MSSM (sMSSM) [9, 10]. Comparisons among these singlet-extended MSSM models can be found in refs. [11]. A common feature in these models is that there is no fundamental μ term in the superpotential. After the symmetry breaking associated with the singlet field S, the μ term is effectively generated by the product of the dimensionless coupling and the vacuum expectation value (VEV) of S, and thus no fine tuning is required. Because of the introduction of singlet field(s), such models have richer physics than the MSSM.

In this paper, we focus on the Higgs sector of the sMSSM with particular emphasis on CP violation. The sMSSM is a string-inspired model whose particle content of the Higgs sector comprises two Higgs doublets and four Higgs singlets. They are charged under the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)'_{Q'}$ gauge symmetry. Once the additional $\mathrm{U}(1)$ symmetry is introduced, a new gauge boson Z' must exist in the model and can mix with the ordinary Z boson [12, 13]. From the negative results of Z' search at LEP, the magnitude of the mixing angle between them (denoted by $\alpha_{ZZ'}$) must be suppressed at $\mathcal{O}(10^{-3})$ level [14]. The sMSSM provides an explanation for such a Z-Z' hierarchy in a natural way. If the $\mathrm{U}(1)'$ symmetry is broken around the TeV scale, the VEVs of the additional three Higgs singlets (S_1, S_2, S_3) are expected to be of $\mathcal{O}(\mathrm{TeV})$. This makes $\alpha_{ZZ'}$ small enough to escape from the current experimental bounds on the Z' boson.

Due to the extension in the Higgs sector, it is possible to break the CP symmetry explicitly and spontaneously at the tree level, which is forbidden in the MSSM. It is well known that the Kobayashi-Maskawa CP-violating phase [15] in the SM is too small to generate sufficiently large baryon asymmetry of the Universe as observed today [16]. Therefore, additional CP-violating phases are required for successful baryogenesis. So far, electroweak baryogenesis have been studied in the singlet extended MSSM models: the NMSSM [17], the nMSSM [5, 18], the UMSSM [19] and the sMSSM [20]. A detailed analysis of the connection between CP violation and baryogenesis, however, is beyond the scope of this paper.

In our analysis, we use the one-loop effective potential that includes contributions from the third-generation quarks and squarks. We search for the parameter space allowed by imposing both theoretical and experimental constraints on the model. Owing to the presence of extra Higgs singlet fields, the tadpole conditions defined by the first derivatives of the Higgs potential do not always give the desired vacuum, $v = 246\,\mathrm{GeV}$. Therefore, we also numerically check whether or not the minimum is located at 246 GeV. We find that the possible region for the electroweak vacuum is quite limited, depending on the model parameters.

In the sMSSM, the only source of physical CP violation at the tree level comes from the relative phase between the soft SUSY breaking masses and the phases of the Higgs fields. We calculate the Higgs boson masses and the couplings between the gauge bosons and Higgs bosons in the cases of explicit CP violation (ECPV) and spontaneous CP violation (SCPV). It is found that due to the new CP-violating phases, the mass of the lightest Higgs boson can be smaller than that of the Z boson. On the other hand, the coupling of the lightest Higgs boson to the Z boson is sufficiently suppressed, similar to the CPX scenario in the MSSM [21 – 23]. Nonetheless, the μ and A parameters are not necessarily large in this model, making the spectrum of SUSY particles different from the CPX scenario.

We also provide a bound on the lightest Higgs boson mass in the case of SCPV. Depending on the mass of charged Higgs bosons, the upper bound can reach above 100 GeV with maximal CP violation.

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Higgs	$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)'_{Q'}$
H_d	$\Big(\mathbf{1}, \mathbf{2}, -1/2, Q_{H_d}\Big)$
H_u	$\Big(\mathbf{1}, \mathbf{2}, 1/2, Q_{H_u}\Big)$
S	$(\mathbf{1},\ \mathbf{1},\ 0,\ Q_S)$
S_1	$(\mathbf{1},\ \mathbf{1},\ 0,\ Q_{S_1})$
S_2	$(\mathbf{1},\ \mathbf{1},\ 0,\ Q_{S_2})$
S_3	$(\mathbf{1},\ \mathbf{1},\ 0,\ Q_{S_3})$

Table 1: Particle content in the Higgs sector of sMSSM

The paper is organized as follows. In section 2, we introduce the model and define the CP-violating phases in a reparametrization invariant way. Theoretical and experimental constraints are studied in section 3. We examine the effects of CP violation on the Higgs boson masses and couplings in section 4. In particular, the explicit CP-violating case is presented in subsection 4.1 and the spontaneous CP-violating case in subsection 4.2. The discussion about electric dipole moments (EDMs) is presented in subsection 4.3. Finally, we summarize the work in section 5. Formulas of the Higgs boson masses are given in appendix A.

2. The model

The particle content in the Higgs sector of sMSSM comprises two Higgs doublets (H_d, H_u) and four Higgs singlets (S, S_1, S_2, S_3) [9]. As listed in table 1, each field is charged under the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)'_{Q'}$ gauge symmetry. Though it is desirable to have $\mathrm{U}(1)'$ charges (Q's) chosen to make the model anomaly free, a complete analysis of anomaly cancellation is beyond the scope of this paper.[1] Neither will we address the gauge coupling unification issue here as it requires the knowledge of full particle spectrum in the model. Instead, we focus exclusively on the Higgs sector. The model which we are considering is extended so that no dimensionful parameter exists in the superpotential $\mathcal{W}$:

$$\mathcal{W} \ni -\epsilon_{ij}\lambda S H_d^i H_u^j - \lambda_S S_1 S_2 S_3\,, \tag{2.1}$$

where λ and λ_S are the dimensionless couplings. Unlike the NMSSM, the $\mathrm{U}(1)'$ symmetry forbids a cubic term in the superpotential which can cause a domain wall problem if the Z_3 symmetry is broken spontaneously. Once the Higgs singlet S develops a VEV, an effective μ term is generated by $\mu_{\text{eff}} = \lambda\langle S\rangle$. Therefore, the scale of μ_{eff} is determined by the soft SUSY breaking terms. In eq. (2.1) only, there is no interaction between the secluded Higgs singlet fields $S_{1,2,3}$ and the two Higgs doublets $H_{u,d}$ and singlet S.

[1]To be anomaly free, exotic chiral supermultiplets are generally required [7, 24, 25]. For our purpose, we assume that they are heavy enough not to affect the phenomenology at the electroweak scale.

The Higgs potential at the tree level is given by the F-, D- and soft SUSY breaking terms:

$$V_0 = V_F + V_D + V_{\rm soft}, \tag{2.2}$$

where each term reads

$$\begin{aligned} V_F &= |\lambda|^2\Big\{|\epsilon_{ij}\Phi_d^i\Phi_u^j|^2 + |S|^2\left(\Phi_d^\dagger\Phi_d + \Phi_u^\dagger\Phi_u\right)\Big\} + |\lambda_S|^2\left(|S_1S_2|^2 + |S_2S_3|^2 + |S_3S_1|^2\right), && (2.3)\\ V_D &= \frac{g_2^2+g_1^2}{8}(\Phi_d^\dagger\Phi_d - \Phi_u^\dagger\Phi_u)^2 + \frac{g_2^2}{2}|\Phi_d^\dagger\Phi_u|^2 \\ &\quad + \frac{g_1'^2}{2}\left(Q_{H_d}\Phi_d^\dagger\Phi_d + Q_{H_u}\Phi_u^\dagger\Phi_u + Q_S|S|^2 + \sum_{i=1}^{3}Q_{S_i}|S_i|^2\right)^2, && (2.4)\\ V_{\rm soft} &= m_1^2\Phi_d^\dagger\Phi_d + m_2^2\Phi_u^\dagger\Phi_u + m_S^2|S|^2 + \sum_{i=1}^{3}m_{S_i}^2|S_i|^2 && (2.5)\\ &\quad -\Big(\epsilon_{ij}\lambda A_\lambda S\Phi_d^i\Phi_u^j + \lambda_S A_{\lambda_S}S_1S_2S_3 + m_{SS_1}^2SS_1 + m_{SS_2}^2SS_2 + m_{S_1S_2}^2S_1^\dagger S_2 + \text{h.c.}\Big). \end{aligned}$$

where g_2, g_1 and g_1' are the SU(2), U(1) and U(1)$'$ gauge couplings, respectively. We will take $g_1' = \sqrt{5/3}g_1$ as motivated by the gauge unification in the simple GUTs. The soft SUSY breaking masses m_{SS_1} and m_{SS_2} are introduced to break the two unwanted global U(1) symmetries. This choice is called Model I, where $Q_S = -Q_{S_1} = -Q_{S_2} = Q_{S_3}/2$ and $Q_{H_d} + Q_{H_u} + Q_S = 0$. Although the other choice dubbed Model II is also possible, we will not pursue it in this paper since there is no room for physical CP-violating phases in the tree-level potential [9]. The secluded sector (S_1, S_2, S_3) can interact with the ordinary ones (H_d, H_u, S) through the g_1' coupling, m_{SS_1} and m_{SS_2}.

In general, the following five parameters can be complex in the Higgs potential:

$$\lambda A_\lambda,\ \lambda_S A_{\lambda_S},\ m_{SS_1}^2,\ m_{SS_2}^2,\ m_{S_1S_2}^2 \in \boldsymbol{C}. \tag{2.6}$$

After rephasing the Higgs fields, however, four of them can be made real and only one CP-violating phase is physical. In the following, we define the CP-violating phase in a reparametrization invariant way. It should be noted that in the UMSSM no physical CP-violating phase can survive after rotating the Higgs fields and, therefore, the CP symmetry cannot be violated in the tree-level Higgs potential. We parameterize the Higgs fields as

$$\Phi_d = e^{i\theta_1}\begin{pmatrix}\frac{1}{\sqrt{2}}(v_d + h_d + ia_d)\\ \phi_d^-\end{pmatrix}, \qquad \Phi_u = e^{i\theta_2}\begin{pmatrix}\phi_u^+\\ \frac{1}{\sqrt{2}}(v_u + h_u + ia_u)\end{pmatrix}, \tag{2.7}$$

$$S = \frac{e^{i\theta_S}}{\sqrt{2}}(v_S + h_S + ia_S), \qquad S_i = \frac{e^{i\theta_{S_i}}}{\sqrt{2}}(v_{S_i} + h_{S_i} + ia_{S_i}), \quad (i = 1-3), \tag{2.8}$$

where $v = \sqrt{v_d^2 + v_u^2} \simeq 246\,\text{GeV}$. The nonzero θ's can break the CP symmetry spontaneously. However, the θ's are not independent. Here we define the four gauge invariant phases by

$$\varphi_1 = \theta_S + \theta_{S_1}, \quad \varphi_2 = \theta_S + \theta_{S_2}, \quad \varphi_3 = \theta_S + \theta_1 + \theta_2, \quad \varphi_4 = \theta_{S_1} + \theta_{S_2} + \theta_{S_3}. \tag{2.9}$$

For later convenience, we also define $\varphi_{12} = -\varphi_1 + \varphi_2$. The first derivative of the Higgs potential with respect to each Higgs field must vanish (tadpole conditions). At the tree level, we obtain

$$\frac{1}{v_d}\left\langle\frac{\partial V_0}{\partial h_d}\right\rangle = m_1^2 + \frac{g_2^2+g_1^2}{8}\left(v_d^2-v_u^2\right) - R_\lambda\frac{v_u v_S}{v_d} + \frac{|\lambda|^2}{2}\left(v_u^2+v_S^2\right) + \frac{g_1'^2}{2}Q_{H_d}\Delta = 0, \tag{2.10}$$

$$\frac{1}{v_u}\left\langle\frac{\partial V_0}{\partial h_u}\right\rangle = m_2^2 - \frac{g_2^2+g_1^2}{8}\left(v_d^2-v_u^2\right) - R_\lambda\frac{v_d v_S}{v_u} + \frac{|\lambda|^2}{2}\left(v_d^2+v_S^2\right) + \frac{g_1'^2}{2}Q_{H_u}\Delta = 0, \tag{2.11}$$

$$\begin{aligned}\frac{1}{v_S}\left\langle\frac{\partial V_0}{\partial h_S}\right\rangle &= m_S^2 - \mathrm{Re}(m_{SS_1}^2 e^{i\varphi_1})\frac{v_{S_1}}{v_S} - \mathrm{Re}(m_{SS_2}^2 e^{i\varphi_2})\frac{v_{S_2}}{v_S} - R_\lambda\frac{v_d v_u}{v_S}\\ &\quad + \frac{|\lambda|^2}{2}(v_d^2+v_u^2) + \frac{g_1'^2}{2}Q_S\Delta = 0,\end{aligned} \tag{2.12}$$

$$\begin{aligned}\frac{1}{v_{S_1}}\left\langle\frac{\partial V_0}{\partial h_{S_1}}\right\rangle &= m_{S_1}^2 - \mathrm{Re}(m_{SS_1}^2 e^{i\varphi_1})\frac{v_S}{v_{S_1}} - \mathrm{Re}(m_{S_1S_2}^2 e^{i\varphi_{12}})\frac{v_{S_2}}{v_{S_1}} - R_{\lambda_S}\frac{v_{S_2}v_{S_3}}{v_{S_1}}\\ &\quad + \frac{|\lambda_S|^2}{2}(v_{S_2}^2+v_{S_3}^2) + \frac{g_1'^2}{2}Q_{S_1}\Delta = 0,\end{aligned} \tag{2.13}$$

$$\begin{aligned}\frac{1}{v_{S_2}}\left\langle\frac{\partial V_0}{\partial h_{S_2}}\right\rangle &= m_{S_2}^2 - \mathrm{Re}(m_{SS_2}^2 e^{i\varphi_2})\frac{v_S}{v_{S_2}} - \mathrm{Re}(m_{S_1S_2}^2 e^{i\varphi_{12}})\frac{v_{S_1}}{v_{S_2}} - R_{\lambda_S}\frac{v_{S_1}v_{S_3}}{v_{S_2}}\\ &\quad + \frac{|\lambda_S|^2}{2}(v_{S_1}^2+v_{S_3}^2) + \frac{g_1'^2}{2}Q_{S_2}\Delta = 0,\end{aligned} \tag{2.14}$$

$$\frac{1}{v_{S_3}}\left\langle\frac{\partial V_0}{\partial h_{S_3}}\right\rangle = m_{S_3}^2 - R_{\lambda_S}\frac{v_{S_1}v_{S_2}}{v_{S_3}} + \frac{|\lambda_S|^2}{2}(v_{S_1}^2+v_{S_2}^2) + \frac{g_1'^2}{2}Q_{S_3}\Delta = 0, \tag{2.15}$$

$$\frac{1}{v_u}\left\langle\frac{\partial V_0}{\partial a_d}\right\rangle = \frac{1}{v_d}\left\langle\frac{\partial V_0}{\partial a_u}\right\rangle = I_\lambda v_S = 0, \tag{2.16}$$

$$\left\langle\frac{\partial V_0}{\partial a_S}\right\rangle = \mathrm{Im}(m_{SS_1}^2 e^{i\varphi_1})v_{S_1} + \mathrm{Im}(m_{SS_2}^2 e^{i\varphi_2})v_{S_2} + I_\lambda v_d v_u = 0, \tag{2.17}$$

$$\left\langle\frac{\partial V_0}{\partial a_{S_1}}\right\rangle = \mathrm{Im}(m_{SS_1}^2 e^{i\varphi_1})v_S - \mathrm{Im}(m_{S_1S_2}^2 e^{i\varphi_{12}})v_{S_2} + I_{\lambda_S}v_{S_2}v_{S_3} = 0, \tag{2.18}$$

$$\left\langle\frac{\partial V_0}{\partial a_{S_2}}\right\rangle = \mathrm{Im}(m_{SS_2}^2 e^{i\varphi_2})v_S + \mathrm{Im}(m_{S_1S_2}^2 e^{i\varphi_{12}})v_{S_1} + I_{\lambda_S}v_{S_1}v_{S_3} = 0, \tag{2.19}$$

$$\left\langle\frac{\partial V_0}{\partial a_{S_3}}\right\rangle = I_{\lambda_S}v_{S_1}v_{S_2} = 0, \tag{2.20}$$

with

$$\Delta = Q_{H_d}v_d^2 + Q_{H_u}v_u^2 + Q_S v_S^2 + \sum_{i=1}^{3} Q_{S_i}v_{S_i}^2, \tag{2.21}$$

$$R_\lambda = \frac{\mathrm{Re}(\lambda A_\lambda e^{i\varphi_3})}{\sqrt{2}}, \qquad I_\lambda = \frac{\mathrm{Im}(\lambda A_\lambda e^{i\varphi_3})}{\sqrt{2}}, \tag{2.22}$$

$$R_{\lambda_S} = \frac{\mathrm{Re}(\lambda_S A_{\lambda_S} e^{i\varphi_4})}{\sqrt{2}}, \qquad I_{\lambda_S} = \frac{\mathrm{Im}(\lambda_S A_{\lambda_S} e^{i\varphi_4})}{\sqrt{2}}, \tag{2.23}$$

where $\langle\cdots\rangle$ is defined such that all Higgs fluctuating fields are taken to be zero. Here all the Higgs VEVs are assumed to be nonzero. For some parameter sets, however, a global minimum can be located at the place where some of the Higgs VEVs are zero. Of course,

	CP-even Higgs bosons	CP-odd Higgs bosons	charged Higgs bosons
CPC	$H_1, H_2, H_3, H_4, H_5, H_6$	A_1, A_2, A_3, A_4	H^+, H^-
CPV	$H_1, H_2, H_3, H_4, H_5, H_6, H_7, H_8, H_9, H_{10}$		H^+, H^-

Table 2: Physical Higgs bosons in the sMSSM

such a minimum cannot be found from eqs. (2.10)–(2.20). We will discuss the method of minimum search in section 3. In the current investigation, we do not specify any SUSY breaking scenario. Hence the soft SUSY breaking masses are given by the tadpole conditions for the CP-even Higgs fields eqs. (2.10)–(2.15). After solving the tadpole conditions for the CP-odd Higgs fields from eqs. (2.16)–(2.20), we find

$$I_\lambda = I_{\lambda_S} = 0, \tag{2.24}$$

$$\mathrm{Im}(m^2_{SS_1} e^{i\varphi_1}) = \mathrm{Im}(m^2_{S_1S_2} e^{i\varphi_{12}}) \frac{v_{S_2}}{v_S}, \tag{2.25}$$

$$\mathrm{Im}(m^2_{SS_2} e^{i\varphi_2}) = -\mathrm{Im}(m^2_{S_1S_2} e^{i\varphi_{12}}) \frac{v_{S_1}}{v_S}. \tag{2.26}$$

The CP-violating phases must satisfy eqs. (2.24)–(2.26) for the vacuum. As a convention, we choose the independent physical CP-violating phase to be $\theta_{\mathrm{phys}} = \mathrm{Arg}(m^2_{S_1S_2}) + \varphi_{12}$.

2.1 The mass matrix of the neutral Higgs bosons

The squared mass matrix of the neutral Higgs bosons is a 12×12 symmetric matrix taking the form

$$\frac{1}{2} \begin{pmatrix} \boldsymbol{H}^T & \boldsymbol{A}^T \end{pmatrix} \mathcal{M}^2_N \begin{pmatrix} \boldsymbol{H} \\ \boldsymbol{A} \end{pmatrix}, \quad \mathcal{M}^2_N = \begin{pmatrix} \mathcal{M}^2_S & \mathcal{M}^2_{\mathrm{SP}} \\ (\mathcal{M}^2_{\mathrm{SP}})^T & \mathcal{M}^2_P \end{pmatrix}, \tag{2.27}$$

where $\boldsymbol{H}^T \equiv (\boldsymbol{h}_O^T = (h_d\ h_u\ h_S)\ \boldsymbol{h}_S^T = (h_{S_1}\ h_{S_2}\ h_{S_3}))$, $\boldsymbol{A}^T \equiv (\boldsymbol{a}_O^T = (a_d\ a_u\ a_S)\ \boldsymbol{a}_S^T = (a_{S_1}\ a_{S_2}\ a_{S_3}))$. The subscripts O and S on $\boldsymbol{h}/\boldsymbol{a}$ denote 'ordinary' and 'secluded', respectively. In table 2, the physical Higgs bosons in this model are listed for both the CP-conserving (CPC) and the CP-violating (CPV) cases. After the symmetry breaking, two neutral Nambu-Goldstone bosons G^0 and G'^0 appear and are absorbed by the Z and Z' bosons, respectively. It is straightforward to decouple G^0 from the squared mass matrix (2.27) analytically by performing the rotation

$$\begin{pmatrix} a_d \\ a_u \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} G^0 \\ a \end{pmatrix}, \tag{2.28}$$

where $\tan\beta \equiv v_u/v_d$. We diagonalize the reduced 11×11 matrix $\tilde{\mathcal{M}}^2_N$ numerically: $O^T \tilde{\mathcal{M}}^2_N O = \mathrm{diag}(m^2_{G'^0}, m^2_1, m^2_2, m^2_3, m^2_4, m^2_5, m^2_6, m^2_7, m^2_8, m^2_9, m^2_{10})$, where $m_i < m_{i+1}$ ($i = 1-9$) and O is an orthogonal matrix. The explicit expressions for the matrix elements in eq. (2.27) at the tree level are presented in appendix A.

A complex $m^2_{S_1S_2}$ and/or a nontrivial φ_{12} can yield nonzero mixing terms between CP-even and CP-odd Higgs bosons:

$$\mathcal{M}^2_{\mathrm{SP}} \propto \mathrm{Im}(m^2_{S_1S_2} e^{i\varphi_{12}}) \ . \tag{2.29}$$

This gives rise to broken CP symmetry. A detailed discussion about the CP-violating effects on the Higgs masses and couplings will be presented in subsections 4.1 and 4.2. In the CP-conserving case, $\mathcal{M}^2_{\rm SP} = \mathbf{0}$ and eq. (2.27) can be decomposed into two 6×6 sub-matrices.

Now we consider the one-loop corrections to the Higgs boson masses. It suffices for the current investigation to take into account the contributions of the third-generation quarks (t, b) and squarks $(\tilde{t}_{1,2}, \tilde{b}_{1,2})$. The one-loop effective potential is given by [26]

$$V_1 = \frac{N_C}{32\pi^2} \sum_{q=t,b} \left[\sum_{a=1,2} \bar{m}^4_{\tilde{q}_a} \left(\ln \frac{\bar{m}^2_{\tilde{q}_a}}{M^2} - \frac{3}{2} \right) - 2\bar{m}^4_q \left(\ln \frac{\bar{m}^2_q}{M^2} - \frac{3}{2} \right) \right] , \tag{2.30}$$

which is regularized using the $\overline{\rm DR}$-scheme. Here N_C denotes the number of colors, $\bar{m}$'s are the background-field-dependent masses, and M is the renormalization scale. We determine M by the condition $\langle V_1 \rangle = 0$, which implies

$$\ln M^2 = \frac{\sum_q [\sum_a m^4_{\tilde{q}_a} \ln m^2_{\tilde{q}_a} - 2m^4_q \ln m^2_q]}{\sum_q [\sum_a m^4_{\tilde{q}_a} - 2m^4_q]} - \frac{3}{2} . \tag{2.31}$$

With the one-loop corrections, the tadpole conditions become

$$0 = \left\langle \frac{\partial V_0}{\partial \phi} \right\rangle + \frac{N_C}{16\pi^2} \sum_{q=t,b} \left[\sum_{a=1,2} \bar{m}^2_{\tilde{q}_a} \left\langle \frac{\partial \bar{m}^2_{\tilde{q}_a}}{\partial \phi} \right\rangle \left(\ln \frac{m^2_{\tilde{q}_a}}{M^2} - 1 \right) - 2m^2_q \left\langle \frac{\partial \bar{m}^2_q}{\partial \phi} \right\rangle \left(\ln \frac{m^2_q}{M^2} - 1 \right) \right] , \tag{2.32}$$

where $m^2 = \langle \bar{m}^2 \rangle$ and ϕ denotes all species of the Higgs fields. The one-loop corrections of the third-generation quarks and squarks to the Higgs boson masses have exactly the same form as in the NMSSM. The explicit formulas can be found in ref. [3],

2.2 The mass matrix of the charged Higgs bosons

The charged Higgs sector is the same as in the MSSM. Once the μ term in the mass formula of the MSSM charged Higgs boson is replaced by the effective μ term, $\mu_{\rm eff} = \lambda v_S e^{i\theta_S}/\sqrt{2}$, we can readily obtain the mass of the charged Higgs bosons in the sMSSM. Its squared mass matrix is given by

$$\begin{pmatrix} \phi^+_d & \phi^+_u \end{pmatrix} \mathcal{M}^2_\pm \begin{pmatrix} \phi^-_d \\ \phi^-_u \end{pmatrix} . \tag{2.33}$$

At the tree level, it follows from eq. (2.33) that

$$m^2_{H^\pm} = \frac{1}{\sin\beta\cos\beta} \left\langle \frac{\partial^2 V_0}{\partial \phi^+_d \partial \phi^-_u} \right\rangle = m^2_W + \frac{2R_\lambda}{\sin 2\beta} v_S - \frac{|\lambda|^2}{2} v^2 . \tag{2.34}$$

Due to the mixing terms between the Higgs doublets and singlets, the relation between the charged Higgs boson mass and the CP-odd Higgs boson mass, $m^2_{H^\pm} = m^2_W + m^2_A$ valid in the MSSM, breaks down in general. In the limit of $\lambda \to 0$ and $v_S \to \infty$ with λv_S being fixed, $m_{SS_1} = m_{SS_2} = 0$ and without CP violation, one of the CP-odd Higgs boson masses is exactly given by $2R_\lambda v_S/\sin 2\beta$. The mass relation in the MSSM is recovered in this particular case.

At the one-loop level, the mass formula of the charged Higgs bosons takes the form [22, 27]

$$\begin{aligned} m_{H^\pm}^2 = m_W^2 + \frac{2R_\lambda v_S}{\sin 2\beta} - \frac{|\lambda|^2}{2}v^2 \\ +\frac{N_C}{16\pi^2\sin\beta\cos\beta}\Bigg[\Bigg(\frac{h(m_{\tilde{t}_1}^2)}{(m_{\tilde{t}_1}^2-m_{\tilde{b}_1}^2)(m_{\tilde{t}_1}^2-m_{\tilde{b}_2}^2)}+\frac{2m_t^2R_tv_S}{v^2\sin^2\beta}\Bigg)f(m_{\tilde{t}_1}^2,m_{\tilde{t}_2}^2) \\ +\Bigg(\frac{h(m_{\tilde{b}_1}^2)}{(m_{\tilde{b}_1}^2-m_{\tilde{t}_1}^2)(m_{\tilde{b}_1}^2-m_{\tilde{t}_2}^2)}+\frac{2m_b^2R_bv_S}{v^2\cos^2\beta}\Bigg)f(m_{\tilde{b}_1}^2,m_{\tilde{b}_2}^2) \\ -\frac{4m_t^2m_b^2}{v^2\sin\beta\cos\beta}f(m_t^2,m_b^2)\Bigg], \end{aligned} \tag{2.35}$$

where $R_{t,b} = \mathrm{Re}(\lambda A_{t,b}e^{i\varphi_3})/\sqrt{2}$, $A_{t,b}$ are defined as the trilinear couplings in the soft SUSY breaking sector, and $f(m_1^2, m_2^2)$ is defined by

$$f(m_1^2,m_2^2) = \frac{1}{m_1^2-m_2^2}\left[m_1^2\left(\ln\frac{m_1^2}{M^2}-1\right)-m_2^2\left(\ln\frac{m_2^2}{M^2}-1\right)\right]. \tag{2.36}$$

The explicit form of $h(m^2)$ is given in ref. [27]. As is done in ref. [3], $|A_\lambda|$ is determined by eq. (2.35). Therefore, we take $m_{H^\pm}$ as an input in our analysis.

3. Allowed region

Finding an acceptable minimum of the Higgs potential is a nontrivial task even at the tree level. Even if we require the tadpole conditions and positive-definiteness of the squared masses of the Higgs bosons, the global minimum can be found at $v \neq 246\,\mathrm{GeV}$. This is because of the presence of the Higgs singlets in the Higgs potential. In ref. [9], the following method is adopted to search for the electroweak vacuum. First, the soft SUSY breaking masses and the two trilinear A terms (A_λ and A_{λ_S}) are taken at arbitrary values. After finding a viable minimum, all the given dimensionful parameters are rescaled so that $v = 246\,\mathrm{GeV}$. In this method, all the Higgs VEVs are determined through the six tadpole conditions (2.10)–(2.15). Therefore unlike the MSSM, $\tan\beta$ is an output. Our method is equivalent to that, but the other way around. Explicitly, we take the Higgs VEVs as the inputs, and then perform the minimum search. That is, $v = 246\,\mathrm{GeV}$ is given in advance. However, as we will see in what follows, the desired electroweak vacuum does not always exist. For some input parameters, the location of $v = 246\,\mathrm{GeV}$ can be unstable and the true minimum would roll down to another point that does not give $v = 246\,\mathrm{GeV}$. Redefining such a minimum as $v = 246\,\mathrm{GeV}$ by rescaling the Higgs VEVs is then inconsistent with the original value of $\tan\beta$ that is scale independent. Therefore, we discard such cases and keep $\tan\beta$ as a fixed input. Before showing the numerical results of the minimum search, we consider theoretical and experimental constraints in the following two subsections, respectively.

JHEP06(2008)019

3.1 Theoretical constraints

The effective potential at the tree level is

$$\begin{aligned}\langle V_0\rangle =\ & \frac{1}{2}m_1^2 v_d^2 + \frac{1}{2}m_2^2 v_u^2 + \frac{1}{2}m_S^2 v_S^2 + \sum_i \frac{1}{2}m_{S_i}^2 v_{S_i}^2 \\ & -\mathrm{Re}(m_{SS_1}^2 e^{i\varphi_1}) v_S v_{S_1} - \mathrm{Re}(m_{SS_2}^2 e^{i\varphi_2}) v_S v_{S_2} - \mathrm{Re}(m_{S_1S_2}^2 e^{i\varphi_{12}}) v_{S_1} v_{S_2}, \\ & -R_\lambda v_d v_u v_S - R_{\lambda_S} v_{S_1} v_{S_2} v_{S_3} + \frac{g_2^2+g_1^2}{32}(v_d^2 - v_u^2)^2 \\ & +\frac{|\lambda|^2}{4}(v_d^2 v_u^2 + v_d^2 v_S^2 + v_u^2 v_S^2) + \frac{|\lambda_S|^2}{4}(v_{S_1}^2 v_{S_2}^2 + v_{S_2}^2 v_{S_3}^2 + v_{S_3}^2 v_{S_1}^2) + \frac{g_1'^2}{8}\Delta^2. \end{aligned} \tag{3.1}$$

In each direction of $v_S = v_{S_1}$ and $v_S = v_{S_2}$ with other VEVs being zero, we demand the coefficients of the quadratic terms be positive so that the effective potential is not unbounded from below:

$$m_S^2 + m_{S_i}^2 - 2\mathrm{Re}(m_{SS_i}^2 e^{i\varphi_i}) > 0\,, \quad i = 1, 2. \tag{3.2}$$

Next we consider the vacuum of the Higgs potential. From the tadpole conditions eqs. (2.10)–(2.20), the vacuum of the tree-level potential takes the form

$$\begin{aligned}\langle V_0\rangle_{\mathrm{vac}} =\ & \frac{1}{2}R_\lambda v_d v_u v_S + \frac{1}{2}R_{\lambda_S} v_{S_1} v_{S_2} v_{S_3} - \frac{g_2^2+g_1^2}{32}(v_d^2 - v_u^2)^2 \\ & -\frac{|\lambda|^2}{4}(v_d^2 v_u^2 + v_d^2 v_S^2 + v_u^2 v_S^2) - \frac{|\lambda_S|^2}{4}(v_{S_1}^2 v_{S_2}^2 + v_{S_2}^2 v_{S_3}^2 + v_{S_3}^2 v_{S_1}^2) - \frac{g_1'^2}{8}\Delta^2 \end{aligned} \tag{3.3}$$

After eliminating R_λ with eq. (2.34) and imposing $\langle V_0\rangle_{\mathrm{vac}} < 0$, the upper bound on the charged Higgs boson mass is obtained:

$$\begin{aligned} m_{H^\pm}^2 <\ & m_W^2 + \frac{2|\lambda|^2 v_S^2}{\sin^2 2\beta} + m_Z^2 \cot^2 2\beta - \frac{4R_{\lambda_S}}{v^2 \sin^2 2\beta} v_{S_1} v_{S_2} v_{S_3} \\ & + \frac{2|\lambda_S|^2}{v^2 \sin^2 2\beta}(v_{S_1}^2 v_{S_2}^2 + v_{S_2}^2 v_{S_3}^2 + v_{S_3}^2 v_{S_1}^2) + \frac{g_1'^2}{v^2 \sin^2 2\beta}\Delta^2 \equiv (m_{H^\pm}^{\max})^2\,. \end{aligned} \tag{3.4}$$

As an example, we plot the maximal value of the charged Higgs boson mass as a function of R_{λ_S} in figure 1. We take $\lambda = -0.8$, $\lambda_S = 0.1$, $v_S = 300\,\mathrm{GeV}$, $v_{S_1} = v_{S_2} = v_{S_3} = 3000\,\mathrm{GeV}$, and $\tan\beta = 1$ (red solid line), 5 (green dotted line), 10 (blue dashed line). The CP-violating phases are assumed to be zero. Since the dominant terms are proportional to $1/\sin^2 2\beta$ in $m_{H^\pm}^{\max}$, $\tan\beta = 1$ gives the smallest $m_{H^\pm}^{\max}$ for a fixed R_{λ_S}. For $R_{\lambda_S} > 0$, the value of $m_{H^\pm}^{\max}$ decreases as R_{λ_S} increases. We find a maximum of $R_{\lambda_S} \simeq 640\,\mathrm{GeV}$.

3.2 Experimental constraints

The $\mathrm{U}(1)'$ charges of the Higgs fields can be constrained by the experimental results of the Z' boson search, namely, the lower bound on the Z' boson mass and the upper bound on the mixing angle between the Z and Z' bosons. The squared mass matrix of the Z and Z' bosons takes the form

$$\mathcal{M}_{ZZ'}^2 = \begin{pmatrix} m_Z^2 & m_Z g_1'(Q_{H_d}\cos^2\beta - Q_{H_u}\sin^2\beta)v \\ m_Z g_1'(Q_{H_d}\cos^2\beta - Q_{H_u}\sin^2\beta)v & m_{Z'}^2 \end{pmatrix}, \tag{3.5}$$

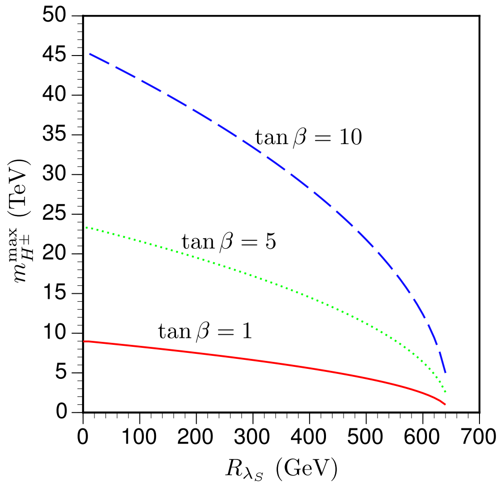

Figure 1: The maximum of charged Higgs boson mass as a function of R_{λ_S}. We take $v_S = 300\,\text{GeV}$, $v_{S_1} = v_{S_2} = v_{S_3} = 3000\,\text{GeV}$, and $\tan\beta = 1$ (red solid line), 5 (green dotted line), 10 (blue dashed line).

where

$$m_Z^2 = \frac{g_2^2 + g_1^2}{4} v^2, \tag{3.6}$$

$$m_{Z'}^2 = g_1'^2 \left(Q_{H_d}^2 v_d^2 + Q_{H_u}^2 v_u^2 + Q_S^2 v_S^2 + \sum_i Q_{S_i}^2 v_{S_i}^2 \right) . \tag{3.7}$$

The eigenvalues of the squared mass matrix and the mixing angle between the Z and Z' bosons are respectively given by

$$m_{Z_{1,2}}^2 = \frac{1}{2} \left[m_Z^2 + m_{Z'}^2 \mp \sqrt{(m_Z^2 - m_{Z'}^2)^2 + g_1'^2 (g_2^2 + g_1^2)(Q_{H_d} v_d^2 - Q_{H_u} v_u^2)^2} \right] , \tag{3.8}$$

$$\alpha_{ZZ'} = \arctan \left(\frac{2 m_Z g_1' (Q_{H_d} \cos^2\beta - Q_{H_u} \sin^2\beta) v}{m_{Z'}^2 - m_Z^2} \right) . \tag{3.9}$$

The experimental constraints on the Z' boson are rather model-dependent. Here we adopt the typical bounds, $m_{Z'} > 600\,\text{GeV}$ and $\alpha_{ZZ'} < \mathcal{O}(10^{-3})$ [14]. In figures 2, we plot the $m_{Z'} = 600\,\text{GeV}$ contour and curves for $\alpha_{ZZ'} = (1, 3, 5) \times 10^{-3}$ in the Q_{H_u}-Q_{H_d} plane. The other $\mathrm{U}(1)'$ charges are determined by the gauge invariance and the condition for breaking the two unwanted global $\mathrm{U}(1)$ symmetries as discussed above. Here we show two examples: (A) $v_S = 300\,\text{GeV}$, $v_{S_1} = v_{S_2} = v_{S_3} = 3000\,\text{GeV}$ with $\tan\beta = 1$ (upper left figure) and $\tan\beta = 50$ (upper right figure); (B) $v_S = 500\,\text{GeV}$, $v_{S_1} = v_{S_3} = 100\,\text{GeV}$, $v_{S_2} = 3000\,\text{GeV}$ with $\tan\beta = 1$ (lower left figure) and $\tan\beta = 10$ (lower right figure). The red dotted lines give the $m_{Z'} = 600\,\text{GeV}$ contour, and the region in between represents $m_{Z'} \leq 600\,\text{GeV}$. The figures also show curves for $\alpha_{ZZ'} = 1 \times 10^{-3}$ (dashed line in green), $\alpha_{ZZ'} = 3 \times 10^{-3}$ (dotted line in blue) and $\alpha_{ZZ'} = 5 \times 10^{-3}$ (solid line in magenta). In the region where Q_{H_d} and Q_{H_u} have the same sign, the two terms in the off-diagonal elements of $\mathcal{M}_{ZZ'}^2$ tend to cancel with each other. The upper right figures show that the $\tan\beta$

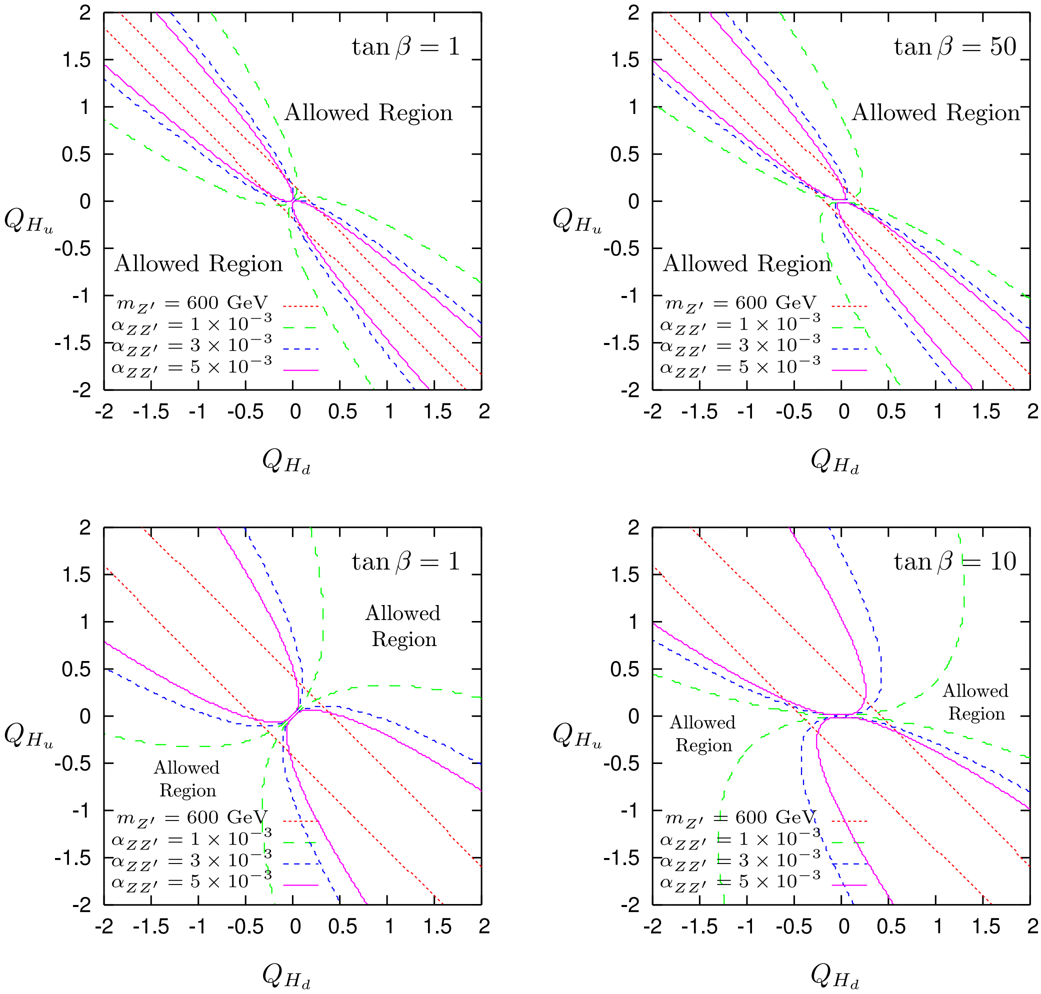

Figure 2: Constraints from the lower bound on $m_{Z'}$ and the upper bound on $\alpha_{ZZ'}$ in the Q_{H_u}-Q_{H_d} plane. We take $v_S = 300\,\text{GeV}$, $v_{S_1} = v_{S_2} = v_{S_3} = 3000\,\text{GeV}$ with $\tan\beta = 1$ (upper left) and $\tan\beta = 50$ (upper right), and $v_S = 500\,\text{GeV}$, $v_{S_1} = v_{S_3} = 100\,\text{GeV}$, $v_{S_2} = 3000\,\text{GeV}$ with $\tan\beta = 1$ (lower left) and $\tan\beta = 10$ (lower right).

dependence on Z' search constraints is rather mild since the denominator in eq. (3.9) is relatively large for case (A). In the lower left figure, the covered areas of quadrants II and IX have $\alpha_{ZZ'} > 1 \times 10^{-3}$. On the other hand, large portions of quadrants I and III are not strongly constrained. If we take $\tan\beta = 10$, the contours of $\alpha_{ZZ'}$ is distorted and the region around $Q_{H_d} \simeq Q_{H_u}/\tan^2\beta$ becomes allowed. In our numerical study, as long as one of v_{S_i} $(i = 1-3)$ is taken to be at the TeV scale and $Q_{H_d} \simeq -Q_{H_u}$ does not hold, the constraints from the Z' boson search can be easily avoided. This supports the original motivation for the sMSSM as mentioned in the Introduction.

According to the LEP experiments, the mass of the SM Higgs boson should be larger than 114.4 GeV at 95 % CL [14]. However, this lower bound cannot be directly applied to models beyond the SM due to the modification of the Higgs coupling to the Z boson

JHEP06(2008)019

(g_{HZZ}). When the Higgs boson masses are smaller than 114.4 GeV, we require instead

$$\xi^2 < k(m_{H_i}) \,, \tag{3.10}$$

where $\xi = g_{HZZ}/g^{\rm SM}_{HZZ}$ and k is the 95 % CL upper limit on the HZZ coupling and a function of the Higgs boson mass [28, 29]. In our analysis, we do not consider the processes $e^+e^- \to Z^* \to H_iH_j$. They are expected to be less severe in comparison with the processes $e^+e^- \to Z^* \to H_iZ$.

We also consider the Z boson decays, $Z \to H_iH_j$ and $Z \to H_il^+l^-$ for the light Higgs bosons, and require that:

$$\sum_{i,j} \Gamma(Z \to H_iH_j) + \sum_i \Gamma(Z \to H_il^+l^-) < \Delta\Gamma_Z \,, \tag{3.11}$$

where $\Delta\Gamma_Z = 2.0$ MeV is the 95 % CL upper bound on the possible additional decay width of the Z boson [30].

The other experimental constraints come from the lower bounds of the SUSY particles. The mass matrix of the charginos has the same form as in the MSSM if we replace μ with $\mu_{\rm eff}$:

$$\mathcal{M}_{\tilde{\chi}^\pm} = \begin{pmatrix} M_2 & -\sqrt{2}m_W\cos\beta \\ -\sqrt{2}m_W\sin\beta & \mu_{\rm eff}e^{i(\theta_1+\theta_2)} \end{pmatrix} , \tag{3.12}$$

where M_2 is the SU(2) gaugino mass. The physical CP-violating phase is $\theta_{M_2} + \theta_\lambda + \varphi_3$, where θ_{M_2} and θ_λ denote the arguments of M_2 and λ, respectively. For the lower bound on the lightest chargino mass $\tilde{\chi}^\pm_1$, we require $m_{\tilde{\chi}^\pm_1} > \sqrt{s}/2 \simeq 104$ GeV, where $\sqrt{s}$ is the center-of-mass energy at LEP2 [31]. On the other hand, the mass bound on the neutralino, $m_{\tilde{\chi}^0} >$ 46 GeV given in ref. [14] is rather model-dependent. In fact, it is found that $m_{\tilde{\chi}^0} \simeq 6$ GeV is allowed in the R-parity conserving MSSM without gaugino mass unification [32]. In the sMSSM, the lightest neutralino can even be massless, almost a singlino [33]. Therefore we will not put an explicit lower bound on the mass of the lightest neutralino, and not require that the lightest neutralino be a candidate for the cold dark matter of the Universe as well.

Now we consider extra contributions to the ρ parameter. It can be easily shown that if a model has only Higgs doublets and singlets, $\rho = 1$ at the tree level. As discussed before, as long as $\alpha_{ZZ'} < \mathcal{O}(10^{-3})$, the deviation of the ρ parameter from unity due to the Z' boson is small enough to evade the current experimental bound $\Delta\rho < 2.0 \times 10^{-3}$ [14]. Let us consider the one-loop corrections, focusing particularly on the contributions of the physical Higgs bosons rather than including all SUSY particles. The correction to the ρ parameter is given by

$$\Delta\rho = \frac{\Pi^T_{ZZ}(0)}{m_Z^2} - \frac{\Pi^T_{WW}(0)}{m_W^2} \,, \tag{3.13}$$

where $\Pi^T_{VV}(0)$ $(V = Z, W)$ are the transverse parts of the weak boson self-energies at the zero momentum. The Higgs boson contributions at the one-loop level take the form

$$\Delta\rho^{\rm Higgs} = \frac{G_F}{8\sqrt{2}\pi^2}\left[\sum_{i<j} g^2_{H_iH_jZ}B_5(m_{H_i}, m_{H_j}) - \sum_i |g_{H_iHW}|^2 B_5(m_{H^\pm}, m_{H_i})\right] , \tag{3.14}$$

with

$$B_5(m_1, m_2) = \begin{cases} -\frac{1}{2}(m_1^2 + m_2^2) + \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \ln \frac{m_1^2}{m_2^2} \quad (m_1 \neq m_2), \\ 0 \quad (m_1 = m_2) \end{cases}, \tag{3.15}$$

$$g_{H_i H_j Z} = (O_{1i} O_{7j} - O_{1j} O_{7i}) \sin\beta - (O_{2i} O_{7i} - O_{2j} O_{7i}) \cos\beta\,, \tag{3.16}$$

$$g_{H_i H W} = O_{2i} \cos\beta - O_{1i} \sin\beta - i O_{7i}, \tag{3.17}$$

where $G_F = 1/(\sqrt{2}v^2) \simeq 1.166 \times 10^{-5}$ $(\mathrm{GeV})^{-2}$. Unlike the MSSM, the custodial SU(2) symmetry does not guarantee $\Delta\rho^{\mathrm{Higgs}} = 0$ due to the contributions from the Higgs singlets.

Finally we comment in passing on the constraints from B physics. The experimental results of $B_s \to \mu^+\mu^-$, $b \to s\gamma$ and $B_u^- \to \tau^-\bar{\nu}_\tau$ can give a significant restriction on the parameter space. However, so long as we limit our interest to the low $\tan\beta$ region ($\lesssim$ 20), constraints from the branching ratios of $B_s \to \mu^+\mu^-$ and $B_u \to \tau\nu_\tau$ are less stringent. The $b \to s\gamma$ process can be important for the light charged Higgs bosons scenario, $m_{H^\pm} \lesssim 300\,\mathrm{GeV}$, in which case the contributions from the charged Higgs bosons and those of the charginos have to cancel [34] in a way to be consistent with the data [35]. We leave the detailed analysis to another paper.

3.3 Numerical evaluation

Now we show the numerical results of the allowed regions in both case I and case II. We take

$$\begin{aligned} &Q_{H_d} = Q_{H_u} = 1, && A_{\lambda_S} = A_\lambda(m_{H^\pm}), && A_t = A_b = \mu_{\mathrm{eff}}/\tan\beta, \\ &m_{\tilde{q}} = 1000 \text{ GeV}, && m_{\tilde{t}_R} = m_{\tilde{b}_R} = 500 \text{ GeV}, && M_2 = 200 \text{ GeV}, \end{aligned} \tag{3.18}$$

where $m_{\tilde{q}}$, $m_{\tilde{t}_R}$ and $m_{\tilde{b}_R}$ are the soft SUSY breaking masses of squarks. It should be noted that A_λ is a function of $m_{H^\pm}$, as given by eq. (2.35). In figure 3, the allowed region is plotted in the λ_S-λ plane (left figure) and $\tan\beta$-$m_{H^\pm}$ plane (right figure). The input parameters in Case I are

$$\begin{aligned} \text{Case I}: \quad & m_{SS_1}^2 = m_{SS_2}^2 = (500 \text{ GeV})^2, && m_{S_1S_2}^2 = -(50 \text{ GeV})^2, \\ & v_S = 300 \text{ GeV}, && v_{S_1} = v_{S_2} = v_{S_3} = 3000 \text{ GeV}. \end{aligned} \tag{3.19}$$

For the moment, all the CP-violating phases are assumed to be zero. In the left figure, we take $\tan\beta = 1$ and $m_{H^\pm} = 300\,\mathrm{GeV}$. All the Higgs boson masses are non-negative in the region between the two blue curves. For fixed λ, the depth of the vacuum decreases as λ_S decreases and eventually becomes higher than the origin, as can been seen from eq. (3.3). The dotted curve in magenta corresponds to the critical situation, below which the vacuum becomes metastable. The region to the right of the dotted-dashed line in green has been excluded by the condition (3.10). Likewise, the region to the right of the dashed line in red is excluded by the chargino lower mass bound. In the right figure, we take $\lambda = -0.8$, $\lambda_S = 0.1$. As in the left figure, $m_H^2 \geq 0$ is fulfilled between the two blue curves, within which the vacuum becomes metastable below the dotted curve in magenta. The region below the dotted-dashed curve in green is excluded by the condition (3.10),

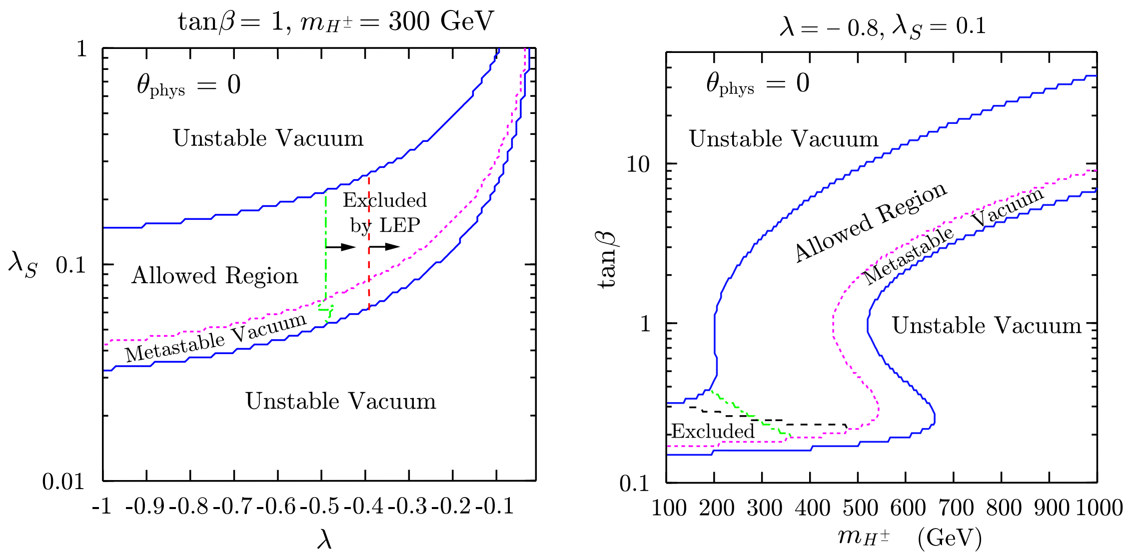

Figure 3: The allowed region in the λ_S-λ plane (left figure) and $\tan\beta$-$m_{H^\pm}$ plane (right figure). We take $Q_{H_d} = Q_{H_u} = 1$, $m^2_{SS_1} = m^2_{SS_2} = (500 \text{ GeV})^2$, $m^2_{S_1S_2} = -(50 \text{ GeV})^2$, $v_S = 300\,\text{GeV}$, $v_{S_1} = v_{S_2} = v_{S_3} = 3000\,\text{GeV}$.

and that below the dashed curve in black by $\Delta\rho > 2.0 \times 10^{-3}$. Since the Higgs singlets can affect the lightest Higgs boson mass, the possibility $\tan\beta = 1$ excluded in the MSSM is experimentally allowed in our model. On the contrary, the allowed region is much more restricted by the conditions for the desired electroweak vacuum.

In figure 4, we consider

$$\begin{aligned} \text{Case II}: \quad & m^2_{SS_1} = (306 \text{ GeV})^2, \quad m^2_{SS_2} = (56 \text{ GeV})^2, \qquad m^2_{S_1S_2} = (100 \text{ GeV})^2, \\ & v_S = 500 \text{ GeV}, \qquad v_{S_1} = v_{S_3} = 100 \text{ GeV}, \qquad v_{S_2} = 3000 \text{ GeV}. \end{aligned} \tag{3.20}$$

In the left figure, we use $\tan\beta = 1$ and $m_{H^\pm} = 600\,\text{GeV}$. The region to the left of the blue line is excluded by $m_H^2 < 0$, and that above the dashed curve in blue results in the situation where $V = V_0 + V_1$ is unbounded from below. In the region between the two lines in magenta, the vacuum is correctly located at $v = 246\,\text{GeV}$. However, the region to the left of the dotted-dashed line in green is excluded by eq. (3.10). The fact that $m_{H^\pm}$ in this case is larger than Case I implies that R_λ is larger. A small λ can make the vacuum metastable, as can be seen from eq. (3.3). In the right figure, we take $\lambda = 0.8$ and $\lambda_S = 0.1$. The allowed region is inside the two dotted-dashed curves in green and the two dashed lines in orange. The dotted-dashed curves in green are obtained from the critical value of the LEP bound (3.10) explained above. The dashed lines in orange correspond to $\alpha_{ZZ'} = 1 \times 10^{-3}$. The parameter space is highly constrained in Case II.

4. *CP* violation

In this section, we study the effects of CP violation in the Higgs sector. In the MSSM, the CP-violating phase in the Higgs potential can be rotated away by a field redefinition. Hence

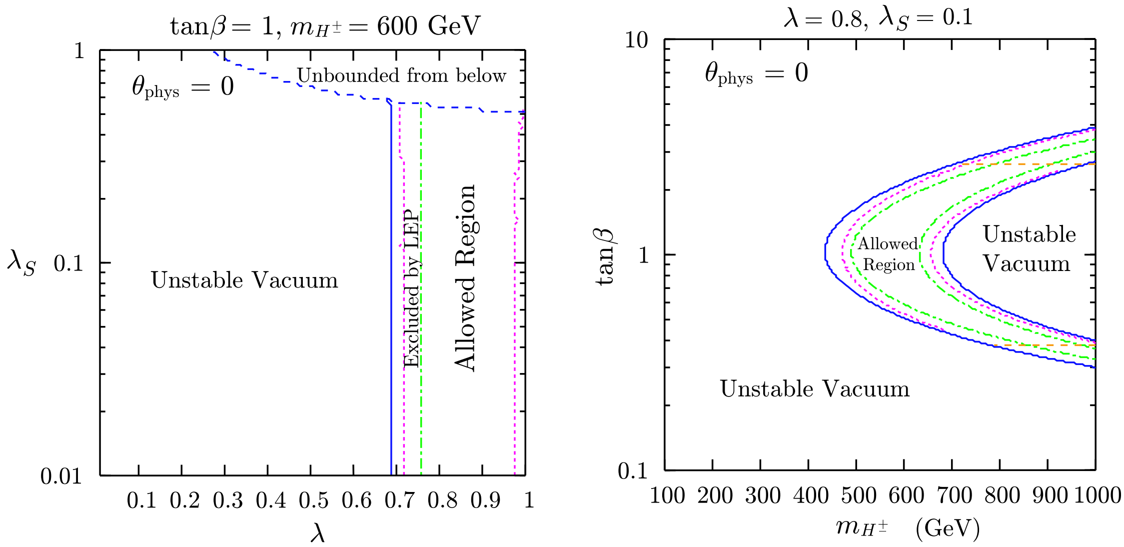

Figure 4: The allowed region in the λ_S-λ plane (left figure) and $\tan\beta$-$m_{H^\pm}$ plane (right figure). We take $Q_{H_d} = Q_{H_u} = 1$, $m^2_{SS_1} = (306\ \text{GeV})^2$, $m^2_{SS_2} = (56\ \text{GeV})^2$, $m^2_{S_1S_2} = (100\ \text{GeV})^2$, $v_S = 500\,\text{GeV}$, $v_{S_1} = v_{S_2} = 100\,\text{GeV}$ and $v_{S_3} = 3000\,\text{GeV}$.

there is no explicit CP violation at the tree level. However, once the one-loop corrections from the squark sector to the Higgs boson masses are taken into account, mixing terms between the CP-even and CP-odd Higgs bosons are generated. In a specific CP-violating case called the CPX scenario, the effects of CP violation is extremely enhanced, and the Higgs phenomenology is drastically changed [21 – 23]. The lightest Higgs boson mass, for example, can become much smaller than the current LEP lower bound due to the large $\mathcal{M}^2_{\rm SP}$ in the squared mass matrix. Its coupling to the Z boson, however, can be sufficiently suppressed to escape from the LEP constraints [29]. Studies of ECPV have been done in the NMSSM [3, 36, 37], nMSSM [5] and the UMSSM [38, 39] as well. Here we discuss both ECPV and SCPV in the sMSSM.

4.1 Explicit CP violation

As discussed in section 2, there is one CP-violating phase that cannot be removed by rephasing the Higgs fields. In fact, the nonzero CP-violating phases are related to each other in the vacuum through the tadpole conditions for the CP-odd Higgs fields. At the one-loop level, we find

$$I_\lambda = -\frac{N_C}{8\pi^2 v^2}\left[\frac{m_t^2 I_t}{\sin^2\beta} f\left(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}\right) + \frac{m_b^2 I_b}{\cos^2\beta} f\left(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}\right)\right], \tag{4.1}$$

$$I_{\lambda_S} = 0, \tag{4.2}$$

$$\text{Im}(m^2_{SS_1} e^{i\varphi_1}) = \text{Im}(m^2_{S_1S_2} e^{i\varphi_{12}})\frac{v_{S_2}}{v_S}, \tag{4.3}$$

$$\text{Im}(m^2_{SS_2} e^{i\varphi_2}) = -\text{Im}(m^2_{S_1S_2} e^{i\varphi_{12}})\frac{v_{S_1}}{v_S}, \tag{4.4}$$

where $I_{t,b} = \text{Im}(\lambda A_{t,b} e^{i\varphi_3})/\sqrt{2}$. If I_t or I_b is nonzero, I_λ can be nonzero as well at the one-loop level. Nevertheless, we will focus exclusively on CP violation peculiar to the sMSSM, and take $I_t = I_b = 0$ in what follows. Since we have the relation eq. (2.35), the sign of R_λ is determined through

$$\mathrm{sgn}(R_\lambda) = \mathrm{sgn}\left(m_{H^\pm}^2 - m_W^2 + \frac{|\lambda|^2}{2} v^2 - \Delta m_{H^\pm}^2 \right), \tag{4.5}$$

where $\Delta m_{H^\pm}^2$ denotes the one-loop correction to the charged Higgs boson mass. On the contrary, there is a sign ambiguity in R_{λ_S} at this stage. The positivity of the squared mass of the Higgs bosons gives us $R_{\lambda_S} > 0$ in most of the parameter space. Now let us define $\theta_{SS_1} = \mathrm{Arg}(m_{SS_1}^2)$, $\theta_{SS_2} = \mathrm{Arg}(m_{SS_2}^2)$, $\theta_{S_1S_2} = \mathrm{Arg}(m_{S_1S_2}^2)$. From eqs. (4.3) and (4.4), it follows that

$$\theta_{SS_1} = \sin^{-1}\left[\left| \frac{m_{S_1S_2}^2}{m_{SS_1}^2} \right| \frac{v_{S_2}}{v_S} \sin(\theta_{S_1S_2} + \varphi_{12}) \right] - \varphi_1, \tag{4.6}$$

$$\theta_{SS_2} = \sin^{-1}\left[- \left| \frac{m_{S_1S_2}^2}{m_{SS_2}^2} \right| \frac{v_{S_1}}{v_S} \sin(\theta_{S_1S_2} + \varphi_{12}) \right] - \varphi_2. \tag{4.7}$$

It should be noted that the arguments in the arcsines should be smaller than one, imposing additional constraints on our input parameters.

The CP-violating phases show up in the mixing terms between CP-even and CP-odd parts in the squared mass matrix (2.27). Let us parameterize $\mathcal{M}_{\mathrm{SP}}^2$ in terms of 3×3 block entries:

$$\frac{1}{2} \begin{pmatrix} \boldsymbol{h}_O^T & \boldsymbol{h}_S^T \end{pmatrix} \mathcal{M}_{\mathrm{SP}}^2 \begin{pmatrix} \boldsymbol{a}_O \\ \boldsymbol{a}_S \end{pmatrix}, \quad \mathcal{M}_{\mathrm{SP}}^2 = \begin{pmatrix} \mathcal{M}_{\mathrm{SP}}^{(O)} & \mathcal{M}_{\mathrm{SP}}^{(OS)} \\ \left(\mathcal{M}_{\mathrm{SP}}^{(OS)}\right)^T & \mathcal{M}_{\mathrm{SP}}^{(S)} \end{pmatrix}. \tag{4.8}$$

After the conditions (4.3) and (4.4) are applied, the entries are

$$\mathcal{M}_{\mathrm{SP}}^{(O)} = \mathbf{0}_{3\times 3}, \quad \mathcal{M}_{\mathrm{SP}}^{(OS)} = \mathrm{Im}(m_{S_1S_2}^2 e^{i\varphi_{12}}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{v_{S_2}}{v_S} & -\frac{v_{S_1}}{v_S} & 0 \end{pmatrix}, \tag{4.9}$$

$$\mathcal{M}_{\mathrm{SP}}^{(S)} = \mathrm{Im}(m_{S_1S_2}^2 e^{i\varphi_{12}}) \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{4.10}$$

If $\mathcal{M}_{\mathrm{SP}}^2$ has a large portion in $\mathcal{M}_N^2$, the CP-violating effects on the Higgs boson masses can be enhanced. To achieve this, we assume large values for $\mathrm{Im}(m_{S_1S_2}^2 e^{i\varphi_{12}}) v_{S_2}/v_S$ and $\mathrm{Im}(m_{S_1S_2}^2 e^{i\varphi_{12}}) v_{S_1}/v_S$ under the conditions (4.6) and (4.7), rendering

$$|m_{SS_1}^2| \simeq |m_{S_1S_2}^2| \frac{v_{S_2}}{v_S}, \tag{4.11}$$

$$|m_{SS_2}^2| \simeq |m_{S_1S_2}^2| \frac{v_{S_1}}{v_S}, \tag{4.12}$$

for $\sin(\theta_{S_1S_2} + \varphi_{12}) \simeq 1$. For the moment, we only consider ECPV, and hence $\varphi_1 = \varphi_2 = 0$. We present two examples: one being Case II as given in eq. (3.20) and the other being

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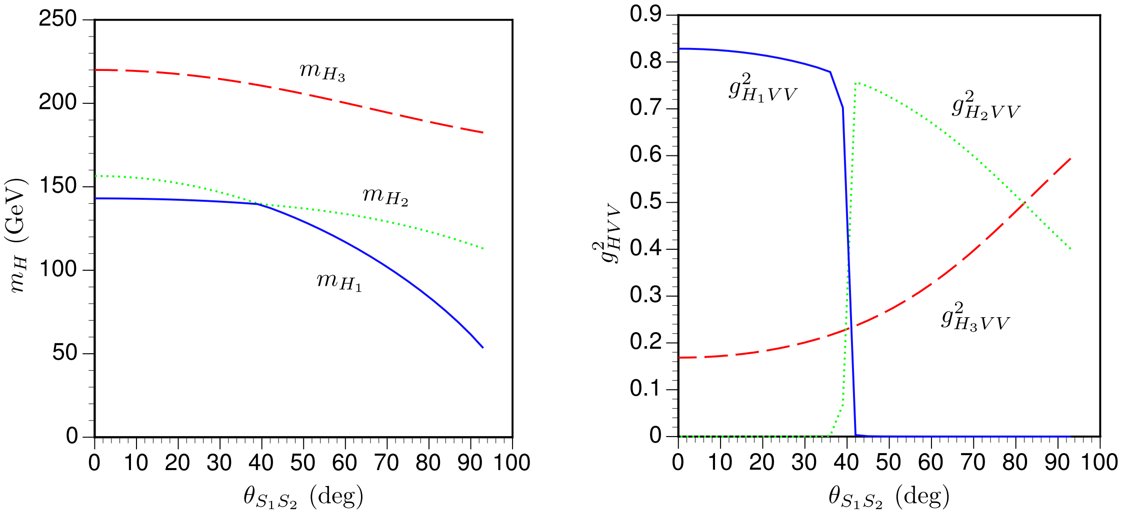

Figure 5: The effects of the CP-violating phase on m_H and g^2_{HVV}. We take $m_{H^\pm} = 600\,\text{GeV}$, $\tan\beta = 1$, $|m^2_{SS_1}| = (306\ \text{GeV})^2$, $|m^2_{SS_2}| = (56\ \text{GeV})^2$, $|m^2_{S_1S_2}| = (100\ \text{GeV})^2$, $v_S = 500$ GeV, $v_{S_1} = v_{S_3} = 100\,\text{GeV}$, and $v_{S_2} = 3000\,\text{GeV}$.

Case III specified by

$$\begin{aligned} \text{Case III}: \quad & m^2_{SS_1} = (72\ \text{GeV})^2, \quad m^2_{SS_2} = (280\ \text{GeV})^2, \qquad m^2_{S_1S_2} = (100\ \text{GeV})^2, \\ & v_S = 300\ \text{GeV}, \qquad v_{S_1} = v_{S_3} = 1500\ \text{GeV}, \qquad v_{S_2} = 100\ \text{GeV}. \end{aligned} \tag{4.13}$$

We take $\tan\beta = 1$ and $m_{H^\pm} = 600\,\text{GeV}$ for Case II and $\tan\beta = 1$ and $m_{H^\pm} = 300\,\text{GeV}$ for Case III. In figure 5, we plot m_{H_i} and $g^2_{H_iVV}$ ($i = 1-3$) as functions of $\theta_{S_1S_2}$ in Case II. In the CP-conserving case, $\theta_{S_1S_2} = 0$, the second lightest Higgs boson is CP-odd because g_{H_2VV} is zero. Around $\theta_{S_1S_2} \simeq 40^\circ$, H_1 and H_2 switch with each other and their CP characters are exchanged, as can be seen from the right figure in figure 5. As in the CP-violating MSSM, due to the large off-diagonal terms $\mathcal{M}^2_{\text{SP}}$, H_1 can become lighter than 114.4 GeV for $\theta_{S_1S_2} \gtrsim 60^\circ$ with $g^2_{H_1VV}$ being highly suppressed. This possibility cannot be excluded by the LEP experimental results. This does not seem to be typical in the CP-violating NMSSM [3]. Although all the Higgs boson masses are positive in the range $93^\circ \lesssim \theta_{S_1S_2} \lesssim 102^\circ$, the vacuum is metastable and is thus excluded. In figure 6, we plot m_{H_i} and $g^2_{H_iVV}$ ($i = 1-3$) as functions of $\theta_{S_1S_2}$ for Case III. When $\theta_{S_1S_2} = 0$, H_1 is the CP-odd Higgs boson since $g_{H_1VV} = 0$. In this parameter set, H_3 is the SM-like Higgs boson, corresponding to the decoupling limit in the MSSM. Both H_1 and H_2 are composed of almost singlet components. The mass m_{H_1} is always smaller than the LEP bound when we vary $\theta_{S_1S_2}$, and can become as low as 20 GeV around $\theta_{S_1S_2} = 102^\circ$. Since $g^2_{H_1VV}$ is less than 10^{-3}, the associated production cross section of H_1 with gauge bosons is highly suppressed. The masses and couplings of the other Higgs bosons are not much affected by CP violation.

4.2 Spontaneous CP violation

In this subsection, we discuss the SCPV scenario. If the model contains two Higgs doublets, one of the Higgs VEVs can be complex in principle. In the MSSM, there is no room for

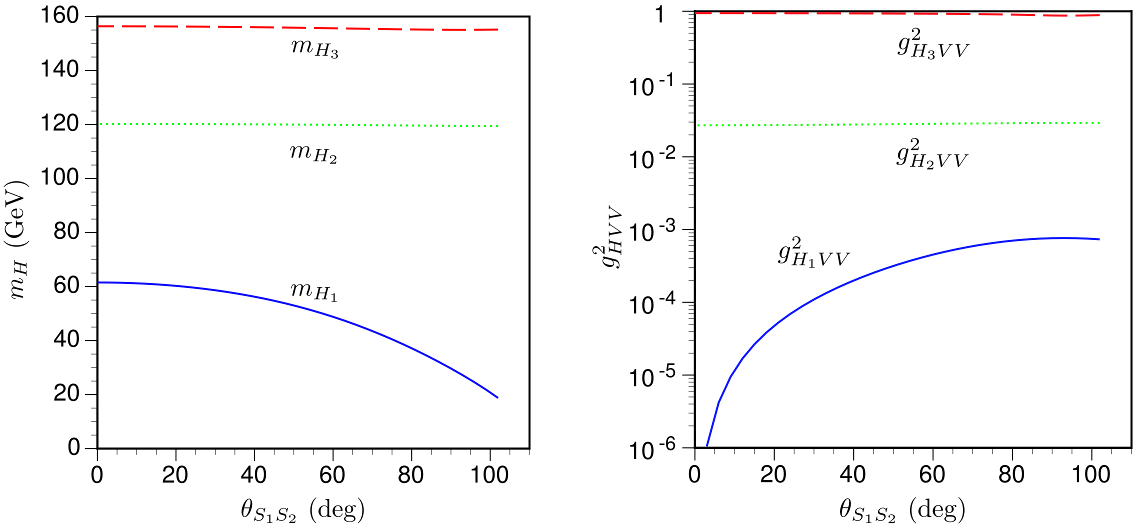

Figure 6: The effects of the CP-violating phase on m_H and g^2_{HVV}. We take $m_{H^\pm} = 300\,\text{GeV}$, $\tan\beta = 1$, $|m^2_{SS_1}| = (72\text{ GeV})^2$, $|m^2_{SS_2}| = (280\text{ GeV})^2$, $|m^2_{S_1S_2}| = (100\text{ GeV})^2$, $v_S = 300\,\text{GeV}$, $v_{S_1} = v_{S_3} = 1500\,\text{GeV}$, and $v_{S_2} = 100\,\text{GeV}$.

the relative phase between the two Higgs doublets in the potential in the SUSY limit due to $\mathrm{U}(1)_{\mathrm{PQ}}$. The only place where the relative phase can show up is the quadratic mixing term between the two Higgs doublets to break the SUSY softly. After imposing the tadpole conditions, such a phase disappears. It is found that the one-loop corrections to the Higgs potential can induce radiative SCPV [40]. However, it leads to the appearance of a light pseudoscalar ($m_A \lesssim 6\,\text{GeV}$), which is already excluded by the LEP experiments. Many studies have already been done for SCPV in the NMSSM with a Z_3 symmetry [41–44]. According to Romão's No-Go theorem [42], with certain radiative corrections in the Higgs sector the condition for SCPV leads to a negative squared-mass mode in the Higgs spectrum. However, it is pointed out by Babu and Barr [43] that the large radiative corrections from the top/stop loops have not been taken into account in the proof of the No-Go theorem. The original saddle point in the Higgs potential can become a minimum in this case and, therefore, the tachyonic mode no longer appears. In ref. [44], the upper bound on the lightest Higgs boson mass is found to be about 140 GeV in the case of SCPV where the full one-loop corrections of top/stop have been included in their calculations. In the NMSSM without a Z_3 symmetry, the No-Go theorem cannot be applied any more. Hence, the SCPV scenario is viable even at the tree level [45].

In the sMSSM, SCPV is induced by the nonzero θ's that appear in the quadratic terms of the Higgs potential. This is also free from the No-Go theorem. To simplify our study, we assume that $m^2_{SS_1}$, $m^2_{SS_2}$, $m^2_{S_1S_2}$, λA_λ and $\lambda_S A_{\lambda_S}$ are all real. From the tadpole conditions (4.1)–(4.4), we find

$$a\sin\varphi_1 + b\sin\varphi_2 = 0, \tag{4.14}$$

$$a\cos\varphi_1 + b\cos\varphi_2 = -\frac{ab}{c}, \tag{4.15}$$

$$\varphi_3 = \varphi_4 = 0, \tag{4.16}$$

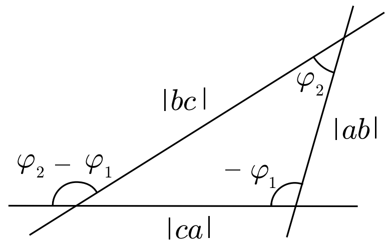

Figure 7: The representative solution for non-zero φ_1 and φ_2.

where $a = m^2_{SS_1} v_S v_{S_1}$, $b = m^2_{SS_2} v_S v_{S_2}$, and $c = m^2_{S_1 S_2} v_{S_1} v_{S_2}$. When eqs. (4.14) and (4.15) have solutions, they form a triangle as depicted in figure 7. The analytic solutions can be easily obtained:

$$\cos\varphi_1 = \frac{1}{2}\left(\frac{bc}{a^2} - \frac{c}{b} - \frac{b}{c}\right), \tag{4.17}$$

$$\cos\varphi_2 = \frac{1}{2}\left(\frac{ac}{b^2} - \frac{a}{c} - \frac{c}{a}\right), \tag{4.18}$$

$$\cos(\varphi_1 - \varphi_2) = \frac{1}{2}\left(\frac{ab}{c^2} - \frac{b}{a} - \frac{a}{b}\right), \tag{4.19}$$

which give the CP-violating extremum. The Higgs potential has the CP-violating minimum when $ac/b < 0$.

We can set $\theta_1 = \theta_{S_3} = 0$ without loss of generality in eq. (2.9). Since $\varphi_3 = \varphi_4 = 0$, it follows that

$$\theta_2 = -\frac{1}{2}(\varphi_1 + \varphi_2), \qquad \theta_S = \frac{1}{2}(\varphi_1 + \varphi_2), \tag{4.20}$$

$$\theta_{S_1} = \frac{1}{2}(\varphi_1 - \varphi_2), \qquad \theta_{S_2} = -\frac{1}{2}(\varphi_1 - \varphi_2). \tag{4.21}$$

We examine the possible maximal value of m_H in the case of SCPV. Since the numerical minimum search is rather time-consuming, we do not conduct a complete parameter scan. Instead, we restrict ourselves to scan only the three soft SUSY breaking masses in the following ranges:

$$\begin{aligned} m^2_{SS_1} &= m^2_{SS_2} = (10\ \mathrm{GeV})^2 - (1000\ \mathrm{GeV})^2, \\ -m^2_{S_1 S_2} &= (1000\ \mathrm{GeV})^2 - (10\ \mathrm{GeV})^2, \end{aligned} \tag{4.22}$$

for fixed values of $m_{H^\pm}$. The remaining parameters are chosen as $\lambda = -0.8$, $\lambda_S = 0.1$, $\tan\beta = 1$, $v_S = 300$ GeV, and $v_{S_1} = v_{S_2} = v_{S_3} = 3000\,\mathrm{GeV}$. In figure 8, the maximal values of m_{H_i} ($i = 1-4$) (left figure) and $|\sin\varphi_1|$ and $|\sin\varphi_2|$ (right figure) are plotted as functions of $m_{H^\pm}$. For each fixed $m_{H^\pm}$, all $m_H^{\max}$ are obtained for different sets of $(m_{SS_1}, m_{SS_2}, m_{S_1 S_2})$. One can see that the upper bounds on m_{H_i} strongly depend on $m_{H^\pm}$ except for m_{H_2}. It is found that the upper bound on the lightest neutral Higgs boson mass $m_{H_1}^{\max}$ is below 125 GeV and can reach up to around 123 GeV for $m_{H^\pm} = 334\,\mathrm{GeV}$. Since the lightest state H_1 is mainly composed of the singlet states, m_{H_1} do not increase even if we change the values of $(m_{\tilde{q}}, m_{\tilde{t}_R}, m_{\tilde{b}_R})$=(1000, 500, 500) GeV to, say (3000, 1500,

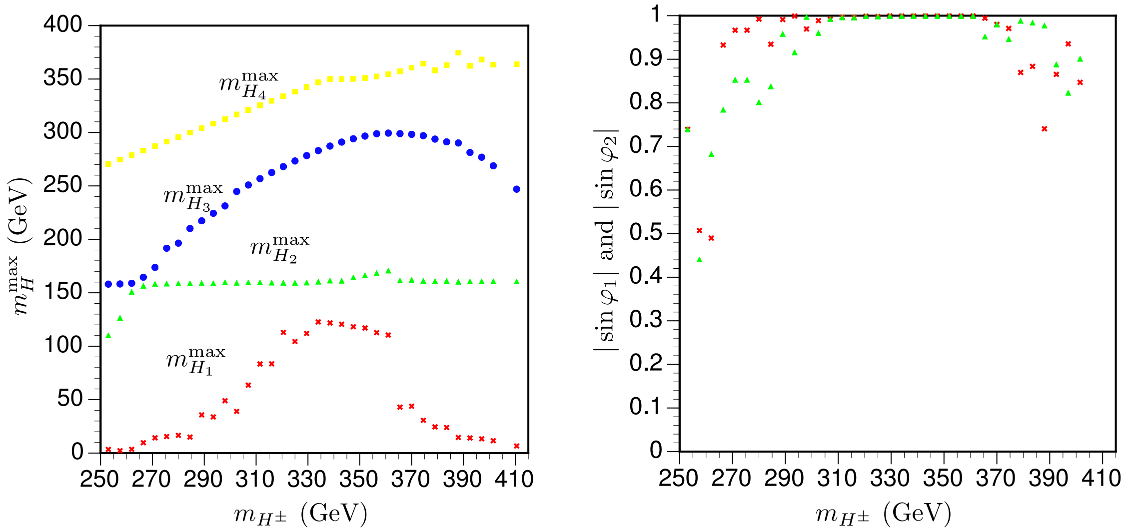

Figure 8: The left plot shows the upper bounds on the four light neutral Higgs boson masses, $m_{H_1}^{\max}$ (cross in red), $m_{H_2}^{\max}$ (triangle in green), $m_{H_1}^{\max}$ (circle in blue) and $m_{H_1}^{\max}$ (square in yellow), as functions of $m_{H^\pm}$. The right plot shows $|\sin\varphi_1|$ and $|\sin\varphi_2|$ in the case of $m_{H_1}^{\max}$. The crosses in red and the triangles in green are for either $|\sin\varphi_1|$ or $|\sin\varphi_2|$.

1500) GeV. In this case, the second lightest Higgs boson H_2 receives corrections from the top/stop loops. In the right plot of figure 8, $|\sin\varphi_1|$ and $|\sin\varphi_2|$ are plotted in the case of $m_{H_1}^{\max}$. The crosses in red and the triangles in green are for either $|\sin\varphi_1|$ or $|\sin\varphi_2|$. One can see that the CP symmetry is maximally violated when $m_{H_1}^{\max} > 100\,\mathrm{GeV}$.

It is noticed that the CP-violating solutions φ_1 and φ_2 are obtained by solving the necessary conditions for SCPV, eqs. (4.14) and (4.15). In order to check whether they give CP violation at the vacuum, we perform the minimization in the ten-dimensional parameter space (v_d, v_u, v_S, v_{S_1}, v_{S_2}, v_{S_3}, θ_2, θ_S, θ_{S_1}, θ_{S_2}), and find that the solutions obtained above indeed give the CP-violating vacuum.

4.3 EDM constraints

The CP-violating phases can also be constrained by the upper bounds on electric dipole moments (EDMs) of electron, neutron and mercury atom [46, 47]. Similar to the MSSM, the SUSY particles-mediated one-loop diagrams contribute to the EDMs. However, we assume that the only sources of CP violation come from $\theta_{S_1S_2}$ for ECPV and φ_i ($i = 1, 2$) for SCPV in the sMSSM. Therefore, their contributions to the EDMs generally vanish. At the two-loop level, however, the Higgs bosons with indefinite CP properties can contribute to the so-called Barr-Zee type diagrams [47] and become sizable when $\tan\beta$ is large. Since we take $\tan\beta = 1$ in the CP-violating cases, we expect that they do not put severe constraints on $\theta_{S_1S_2}$ or φ_i ($i = 1, 2$).

5. Conclusions

We have studied the Higgs sector of the sMSSM with particular focus on CP violation.

The masses and couplings of the Higgs bosons are calculated using the one-loop effective potential, including corrections due to the third-generation quarks and squarks. Imposing both the theoretical and experimental constraints, the allowed region is obtained for Case I and Case II defined in the text. In short, all Higgs VEVs of the secluded Higgs singlets in Case I are taken to be of $\mathcal{O}$(TeV), and in Case II two of them are of $\mathcal{O}(100\,\text{GeV})$ and the other of $\mathcal{O}$(TeV). Due to the corrections from the Higgs singlets, the $\tan\beta = 1$ case cannot be ruled out by the LEP experimental results. However, the conditions for the desired electroweak vacuum generally render a very restrictive parameter space.

In this model, ECPV can be induced by the nonzero phase of $m^2_{S_1S_2}$ at the tree level. It is found that a large value of $\theta_{S_1S_2}$ can make the lightest Higgs boson lighter than the LEP bound of 114.4 GeV, provided that the Higgs coupling to the Z boson is sufficiently suppressed, similar to the CPX scenario in the MSSM. Nevertheless, large μ and A terms are not required in the sMSSM for the realization of large CP violation. Therefore, the spectrum of SUSY particles is generally different from the MSSM CPX scenario.

We have also investigated the SCPV scenario. Unlike the MSSM, SCPV can occur at the tree level in the presence of the nonzero θ's residing in the quadratic terms of the Higgs potential. Our analysis shows that in this case the lightest Higgs boson mass has a certain upper bound, depending on the charged Higgs boson mass. In a specific case, the maximal value of m_{H_1} is around 125 GeV for $m_{H^\pm} = 334$ GeV with the CP-violating phases being nearly maximal.

In this paper, it is assumed that the only sources of CP violation come from the Higgs sector. Such CP-violating phases show up in the Higgs boson-mediated two-loop diagrams that contribute to the EDMs of electron, neutron and mercury atom. However, these diagrams are not important as long as $\tan\beta = 1$.

As pointed out in ref. [20], a strong first order electroweak phase transition is possible in the sMSSM due to the presence of the trilinear term $\lambda A_\lambda S\Phi_d\Phi_u$. In this case, the light stop is not necessarily lighter than the top quark as required in the MSSM. A devoted study of the electroweak phase transition with/without CP violation will be presented elsewhere [48].

A. The mass matrix of the neutral Higgs bosons at the tree level

Here we present explicitly the tree-level squared mass matrix elements for the neutral Higgs bosons. The CP-even part is given by

$$\frac{1}{2}\left(\boldsymbol{h}_O^T\ \boldsymbol{h}_S^T\right)\mathcal{M}_S^2\begin{pmatrix}\boldsymbol{h}_O\\ \boldsymbol{h}_S\end{pmatrix}, \quad \mathcal{M}_S^2 = \begin{pmatrix}\mathcal{M}_S^{(O)} & \mathcal{M}_S^{(OS)}\\ \left(\mathcal{M}_S^{(OS)}\right)^T & \mathcal{M}_S^{(S)}\end{pmatrix}, \tag{A.1}$$

where

$$(\mathcal{M}_S^{(O)})_{11} = \left[\frac{g_2^2+g_1^2}{4} + g_1'^2 Q_{H_d}^2\right]v_d^2 + R_\lambda\frac{v_u v_S}{v_d}, \tag{A.2}$$

$$(\mathcal{M}_S^{(O)})_{22} = \left[\frac{g_2^2+g_1^2}{4} + g_1'^2 Q_{H_u}^2\right]v_u^2 + R_\lambda\frac{v_d v_S}{v_u}, \tag{A.3}$$

$$(\mathcal{M}_S^{(O)})_{33} = \mathrm{Re}(m_{SS_1}^2 e^{i\varphi_1})\frac{v_{S_1}}{v_S} + \mathrm{Re}(m_{SS_2}^2 e^{i\varphi_2})\frac{v_{S_2}}{v_S} + R_\lambda \frac{v_d v_u}{v_S} + g_1'^2 Q_S^2 v_S^2, \tag{A.4}$$

$$(\mathcal{M}_S^{(O)})_{12} = (\mathcal{M}_S^{(O)})_{21} = \left[-\frac{g_2^2+g_1^2}{4} + |\lambda|^2 + g_1'^2 Q_{H_d} Q_{H_u}\right] v_d v_u - R_\lambda v_S, \tag{A.5}$$

$$(\mathcal{M}_S^{(O)})_{13} = (\mathcal{M}_S^{(O)})_{31} = -R_\lambda v_u + (|\lambda|^2 + g_1'^2 Q_{H_d} Q_S) v_d v_S, \tag{A.6}$$

$$(\mathcal{M}_S^{(O)})_{23} = (\mathcal{M}_S^{(O)})_{32} = -R_\lambda v_d + (|\lambda|^2 + g_1'^2 Q_{H_u} Q_S) v_u v_S, \tag{A.7}$$

$$(\mathcal{M}_S^{(S)})_{11} = \mathrm{Re}(m_{SS_1}^2 e^{i\varphi_1})\frac{v_S}{v_{S_1}} + \mathrm{Re}(m_{S_1S_2}^2 e^{i\varphi_{12}})\frac{v_{S_2}}{v_{S_1}} + R_{\lambda_S}\frac{v_{S_2}v_{S_3}}{v_{S_1}} + g_1'^2 Q_{S_1}^2 v_{S_1}^2, \tag{A.8}$$

$$(\mathcal{M}_S^{(S)})_{22} = \mathrm{Re}(m_{SS_2}^2 e^{i\varphi_2})\frac{v_S}{v_{S_2}} + \mathrm{Re}(m_{S_1S_2}^2 e^{i\varphi_{12}})\frac{v_{S_1}}{v_{S_2}} + R_{\lambda_S}\frac{v_{S_1}v_{S_3}}{v_{S_2}} + g_1'^2 Q_{S_2}^2 v_{S_2}^2, \tag{A.9}$$

$$(\mathcal{M}_S^{(S)})_{33} = R_{\lambda_S}\frac{v_{S_1}v_{S_2}}{v_{S_3}} + g_1'^2 Q_{S_3}^2 v_{S_3}^2, \tag{A.10}$$

$$(\mathcal{M}_S^{(S)})_{12} = (\mathcal{M}_S^{(S)})_{21} = -\mathrm{Re}(m_{S_1S_2}^2 e^{i\varphi_{12}}) - R_{\lambda_S} v_{S_3} + (|\lambda_S|^2 + g_1'^2 Q_{S_1} Q_{S_2}) v_{S_1} v_{S_2}, \tag{A.11}$$

$$(\mathcal{M}_S^{(S)})_{13} = (\mathcal{M}_S^{(S)})_{31} = -R_{\lambda_S} v_{S_2} + (|\lambda_S|^2 + g_1'^2 Q_{S_1} Q_{S_3}) v_{S_1} v_{S_3}, \tag{A.12}$$

$$(\mathcal{M}_S^{(S)})_{23} = (\mathcal{M}_S^{(S)})_{32} = -R_{\lambda_S} v_{S_1} + (|\lambda_S|^2 + g_1'^2 Q_{S_2} Q_{S_3}) v_{S_2} v_{S_3}, \tag{A.13}$$

$$(\mathcal{M}_S^{(OS)})_{11} = g_1'^2 Q_{H_d} Q_{S_1} v_d v_{S_1}, \tag{A.14}$$

$$(\mathcal{M}_S^{(OS)})_{22} = g_1'^2 Q_{H_u} Q_{S_2} v_u v_{S_2}, \tag{A.15}$$

$$(\mathcal{M}_S^{(OS)})_{33} = g_1'^2 Q_S Q_{S_3} v_S v_{S_3}, \tag{A.16}$$

$$(\mathcal{M}_S^{(OS)})_{12} = g_1'^2 Q_{H_d} Q_{S_2} v_d v_{S_2}, \tag{A.17}$$

$$(\mathcal{M}_S^{(OS)})_{13} = g_1'^2 Q_{H_d} Q_{S_3} v_d v_{S_3}, \tag{A.18}$$

$$(\mathcal{M}_S^{(OS)})_{21} = g_1'^2 Q_{H_u} Q_{S_1} v_u v_{S_1}, \tag{A.19}$$

$$(\mathcal{M}_S^{(OS)})_{23} = g_1'^2 Q_{H_u} Q_{S_3} v_u v_{S_3}, \tag{A.20}$$

$$(\mathcal{M}_S^{(OS)})_{31} = -\mathrm{Re}(m_{SS_1}^2 e^{i\varphi_1}) + g_1'^2 Q_S Q_{S_1} v_S v_{S_1}, \tag{A.21}$$

$$(\mathcal{M}_S^{(OS)})_{32} = -\mathrm{Re}(m_{SS_2}^2 e^{i\varphi_2}) + g_1'^2 Q_S Q_{S_2} v_S v_{S_2}. \tag{A.22}$$

The CP-odd part is given by

$$\frac{1}{2}\begin{pmatrix} \boldsymbol{a}_O^T & \boldsymbol{a}_S^T \end{pmatrix} \mathcal{M}_P^2 \begin{pmatrix} \boldsymbol{a}_O \\ \boldsymbol{a}_S \end{pmatrix}, \quad \mathcal{M}_P^2 = \begin{pmatrix} \mathcal{M}_P^{(O)} & \mathcal{M}_P^{(OS)} \\ \left(\mathcal{M}_P^{(OS)}\right)^T & \mathcal{M}_P^{(S)} \end{pmatrix}, \tag{A.23}$$

where

$$\mathcal{M}_P^{(O)} = \begin{pmatrix} R_\lambda \frac{v_u v_S}{v_d} & R_\lambda v_S & R_\lambda v_u \\ R_\lambda v_S & R_\lambda \frac{v_d v_S}{v_u} & R_\lambda v_d \\ R_\lambda v_u & R_\lambda v_d & (\mathcal{M}_P^{(O)})_{33} \end{pmatrix}, \quad \mathcal{M}_P^{(OS)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \mathrm{Re}(m_{SS_1}^2 e^{i\varphi_1}) & \mathrm{Re}(m_{SS_2}^2 e^{i\varphi_2}) & 0 \end{pmatrix},$$

$$\mathcal{M}_P^{(S)} = \begin{pmatrix} (\mathcal{M}_P^{(S)})_{11} & -\mathrm{Re}(m_{S_1S_2}^2 e^{i\varphi_{12}}) + R_{\lambda_S} v_{S_3} & R_{\lambda_S} v_{S_2} \\ -\mathrm{Re}(m_{S_1S_2}^2 e^{i\varphi_{12}}) + R_{\lambda_S} v_{S_3} & (\mathcal{M}_P^{(S)})_{22} & R_{\lambda_S} v_{S_1} \\ R_{\lambda_S} v_{S_2} & R_{\lambda_S} v_{S_1} & R_{\lambda_S}\frac{v_{S_1}v_{S_2}}{v_{S_3}} \end{pmatrix}, \tag{A.24}$$

with

$$(\mathcal{M}_P^{(O)})_{33} = \mathrm{Re}(m_{SS_1}^2 e^{i\varphi_1})\frac{v_{S_1}}{v_S} + \mathrm{Re}(m_{SS_2}^2 e^{i\varphi_2})\frac{v_{S_2}}{v_S} + R_\lambda \frac{v_d v_u}{v_S}, \tag{A.25}$$

JHEP06(2008)019

$$(\mathcal{M}_P^{(S)})_{11} = \mathrm{Re}(m_{SS_1}^2 e^{i\varphi_1})\frac{v_S}{v_{S_1}} + \mathrm{Re}(m_{S_1S_2}^2 e^{i\varphi_{12}})\frac{v_{S_2}}{v_{S_1}} + R_{\lambda_S}\frac{v_{S_2}v_{S_3}}{v_{S_1}}, \quad (A.26)$$

$$(\mathcal{M}_P^{(S)})_{22} = \mathrm{Re}(m_{SS_2}^2 e^{i\varphi_2})\frac{v_S}{v_{S_2}} + \mathrm{Re}(m_{S_1S_2}^2 e^{i\varphi_{12}})\frac{v_{S_1}}{v_{S_2}} + R_{\lambda_S}\frac{v_{S_1}v_{S_3}}{v_{S_2}}. \quad (A.27)$$

The mixing between CP-even and CP-odd parts is already given in the main text.

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